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# Use of auctions in allocation of radio spectrum

Master's Thesis
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Spring, 2019
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**Abstract of master's thesis**

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**Title of the thesis:** Use of auctions in allocation of radio spectrum

**Degree:** Master of Science in Economics and Business Administration

**Degree programme:** Master's programme in Economics

**Thesis advisor:** Prof. Pauli Murto

**Year of approval:** 2019 **Number of pages:** 54+6 **Language:** English

## Abstract

The advancement of wireless applications and emergence of new technologies over the past decades has increased the demand for spectrum licenses. This has also increased the complementary nature of licenses reducing the effectiveness of auction mechanisms previously used in spectrum allocation. This especially affects the popular SMRA model which has produced good results in the past, but has poor ability to deal with complementary products. Therefore new auction mechanisms must be developed.

This thesis focuses on theoretical analysis of the most prominent spectrum auction model called the combinatorial clock auction model. CCA model's biggest advantage over other models is its ability to deal with complementary licenses which eliminates the exposure risk and helps the government to reach efficient allocation of radio spectrum. Due to the complexity of the model, the previously popular SMRA and ascending clock auction models will have their applications in the future, but CCA is likely to outperform them in more complex settings. Empirical results show that CCA is producing good outcomes but as the number of licenses grows the auction rules require tweaking in order to maintain the high efficiency.

*Keywords: Spectrum auction, multi-unit auction, combinatorial clock auction, SMRA model*

**Tekijä:** Kalle Piiroinen

**Työn nimi:** Use of auctions in allocation of radio spectrum

**Tutkinto:** Kauppatieteen maisteri

**Koulutusohjelma:** Taloustieteen maisteriohjelma

**Työn ohjaaja:** Prof. Pauli Murto

**Hyväksymisvuosi:** 2019 **Sivumäärä:** 54+6 **Kieli:** Englanti

## Tiivistelmä

Langattoman teknologian kehitys ja applikaatioiden yleistyminen on kasvattanut radiotaajuuksien käyttölupien kysyntää merkittävästi. Samalla komplementaarisuudet eri lisenssien ja taajuuskaistojen välillä ovat vahvistuneet, heikentäen aikaisemmin käytettyjen huutokauppamallien tehokkuutta. Eritoten aikaisemmin yleispätevä SMRA-malli on kohdannut suuria haasteita, koska sen kyky johtaa komplementaaristen lisenssien tehokkaaseen allokaatioon on heikko. Tämä on lisännyt tarvetta uusille huutokauppamalleille, joiden avulla radiotaajuudet voidaan allokoida aikaisempaa tehokkaammin. Tämä työ keskittyy tällä hetkellä lupaavimman huutokauppamallin, kombinatorisen kellohuutokaupan teoreettiseen analyysin. Kombinatorisen kellohuutokaupan suurin vahvuus on sen kyky toimia tehokkaasti, kun lisenssien välillä on komplementaarisuuksia. Tämän myötä huutokaupan voittajien riski voittaa vain osa haluamistaan lisensseistä häviää ja valtion kyky allokoida lisenssit tehokkaasti paranee. Huutokauppamallin monimutkaisuudesta johtuen aikaisemmin käytetyt mallit, kuten SMRA ja tavallinen kellohuutokauppa, tulevat olemaan tärkeässä roolissa tulevaisuudessakin, mutta kombinatorinen kellohuutokauppamalli todennäköisesti johtaa parempiin lopputuloksiin, kun lisenssien välillä on vahvoja komplementaarisuuksia. Käytännön tulokset ovat osoittaneet että kombinatorinen kellohuutokauppa tuottaa hyviä tuloksia, mutta lisenssien lukumäärän kasvaessa huutokaupan säädökset tulevat vaatimaan kehitystä.

*Avainsanat: Taajuushuutokauppa, useiden tuotteiden huutokauppa, kombinatorinen kellohuutokauppa, SMRA-malli*

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# 1 Introduction

Most people have heard the term electromagnetic waves or might even know what they are, but rarely do people comprehend how meaningful part they play in our everyday life. These immaterial signals that fill the air and pass through our bodies non-stop enable us to use the Internet, watch television, listen to the radio, and make phone calls to the other side of the world. All these applications utilize electromagnetic waves that are part of *radio spectrum*, which consists of frequencies between 9 Hz and 300 GHz. The use of the radio spectrum is managed by the international telecommunications union (ITU), which determines the frequencies used for different applications such as mobile communication, civil aviation, military use, and satellite communications. Figure 1 shows an excerpt of the frequency allocation between 470 MHz and 1 GHz.[1]

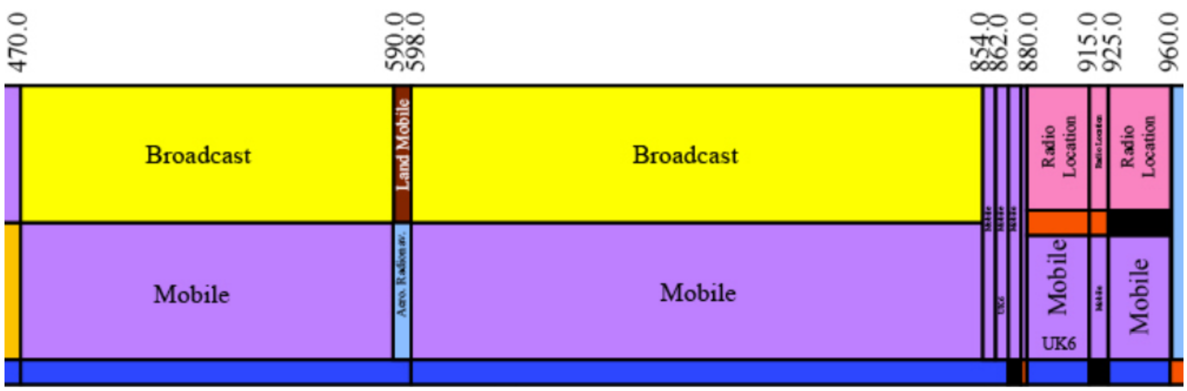


Figure 1: Frequency allocation plan in 470 MHz - 1 GHz range.

In many cases, especially with frequencies that are used for commercial services such as mobile communications, the spectrum use is further regulated by the local governments that award licenses allowing specific parties to use the frequencies. This process is generally referred to as *spectrum allocation* and is in many ways comparable to the process of awarding rights to exploit other national resources, such as minerals or oil. While the available frequency range is seemingly large, only lower part of the spectrum is useful for long-range communication making the allocation of spectrum rights an important topic for the governments.

[1]Full allocation plan is published by the ECC and is available at https://www.ecodocdb.dk/download/2ca5fcbd-4090/ERCREP025.pdf

Before 1990s only few services, such as radio and TV broadcasting, utilized the radio spectrum and the allocation of spectrum rights presented few issues to the governments. However, over the past decades the emergence of countless new wireless technologies and applications has increased the demand for spectrum rights exponentially increasing the value of the radio spectrum as well as transforming it from an abundand resource to a scarce one. The advancement of technology also constantly changes the spectrum allocations plans. For example, many of the frequencies that are used for mobile communications today used to be allocated for analog television broadcasting.
Allocation of spectrum rights, just like any other scarce public resource, is an important topic as it has a direct effect on social welfare. When distributing licenses the first priority should always be efficient allocation of the radio spectrum. Efficient allocation means that the licenses are not just awarded to the party that offers to pay the highest price, but rather the companies that end up with licenses should have the means and resources to provide the best possible services to the consumers. The second priority should be to promote competition in the end market to make sure that the services are available to the consumers at a reasonable cost. Moreover, competition encourages companies to innovate and promote technological advancement. Governments should also try to maximize revenue but only subject to the other objectives.(Binmore and Klemperer, 2002)

Until early 1990s the spectrum rights were often awarded using beauty contests where the companies submitted plans for how they would use the spectrum if they were awarded the rights, and the government then allocated the spectrum based on those plans. This method is very opaque and therefore highly susceptible to favoritism and corruption. Another relatively common allocation method was simply organizing a lottery among the interested companies. Lotteries are fair and transparent, but there are no guarantees that the spectrum rights will be utilized effectively as it is unlikely that the company that values the spectrum most wins the lottery. As the wireless technologies started to develop and new applications emerge, the old allocation methods were not sufficient anymore, and governments started to look for alternative solutions, mainly auctions. Even though the idea of allocating spectrum rights using auctions dates back to the 1950s (Coase, 2013), the first spectrum auction did not take place until 1994 when the Federal Communications Commission (FCC) used a simultaneous ascending auction to distribute phone licenses in the US. Ever since, auctions have been the main tool used by the governments around the world when it comes to allocation of radio spectrum. (Cramton, 2013)

The goal of this paper is to first introduce the reader to spectrum auctions and cover the most commonly used auction models to date, as well as their biggest shortcomings. The main focus of the paper is the combinatorial clock auction model which is the most promising auction model to overcome the issues that have been faced in past auctions. The paper is structured in the following way: First the most important characteristics of spectrum auctions are discussed as well as the most important goals the government should keep in mind when designing spectrum auctions. After that the discussion shifts to the auction models: a short theoretical analysis of the most common multi-unit auction models is presented, which is followed by a discussion covering the common auction models used in spectrum allocation as well as the most common issues governments have faced when allocating the spectrum licenses. Chapter 5 provides a detailed analysis of the combinatorial clock auction model, including its strengths and weaknesses, and a short empirical analysis. Finally, Chapter 6 concludes the paper.

# 2 Spectrum auctions

Empirical results over the past decades have shown that the auctions are a far more efficient mechanism for spectrum allocation than lotteries or beauty contests. One of the most important aspects of auctions is price discovery. In a beauty contest where companies present their business plans they have little incentive to reveal their true valuation for the spectrum, but a well-designed auction almost forces companies to reveal that information, which helps governments to award the licenses to the companies that value the spectrum most. Auctions are also quicker and cheaper to organize than beauty contests, which require a significant amount of time and resources from both parties, the operators and the government. Moreover, auctions have a set of predetermined rules that must be followed, which makes auctions transparent to all the competitors, as well as the public, so no party will be left wondering why they lost, or whether or not there was any foul play. Lastly, auctions can raise significant revenues to the government that can be used improve the wellbeing of the consumers. However, even though it has been more than two decades since the introduction of spectrum auctions, there is still no general consensus on the best auction model for allocation of spectrum rights.(McMillan, 1995; Binmore and Klemperer, 2002; Bichler and Goeree, 2017)

This chapter describes the common characteristics related to spectrum auctions and takes a closer look at the goals governments should be looking to achieve when organizing spectrum auctions.

## 2.1 General characteristics

Planning a spectrum auction usually start with a certain frequency block being reserved for telecommunication services. The frequencies may have been previously used for other purposes or in some cases the advancement of technology has created demand for frequencies that had no real use in the past. The frequency range is then divided into several smaller blocks that are auctioned to the buyers. As a result each winner of the auction has a frequency range reserved only for their use. Figure 2 shows an example of the division and final allocation of the 700 MHz frequencies in Finland after the licenses were auctioned in 2016.

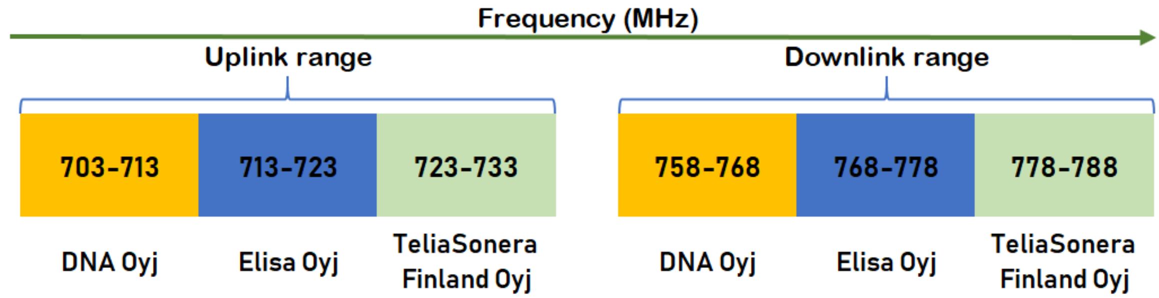


Figure 2: Spectrum allocation in 700 MHz range in Finland.

As in the figure, frequencies are often paired for technological reasons: one part of the frequency pair is used as an uplink while the other is used as a downlink. For operators it would be highly beneficial if both parts of a frequency pair were adjacent to each other and formed a single wider block of frequency as this can reduce the technological challenges considerably when developing the services. This is an important consideration for the auctioneer in cases where the uplink range and downlink range form one continuous block: if the frequencies are paired in a way that one pair is significantly more valuable than others, the strongest operator, i.e., the one with the largest budget, would gain a big advantage over other operators, which might hurt the competitive environment in the end market. On the other hand, pairing the frequencies in another way will lead to lower prices and reduce the revenue the government collects from the auction.

Another aspect the government must consider when dividing the spectrum, and one that truly sets spectrum auctions apart from most multi-item auctions, is the possible existence of complementarities.[2] Spectrum blocks can be divided into two classes: substitutes and complements. If two blocks are substitutes, the bidders' valuation is the same for both spectrum block and they are indifferent to which product they win. In a

[2]Complementarity of products is not limited to spectrum auctions, or even auctions in general, but they are an important consideration in, e.g., sale of franchising rights or forest sell-offs as well. (Levin, 1997).

case like this the auction design becomes a relatively simple problem to solve. However, in most cases there are complementarities across spectrum blocks, which means that the value of one license depends on the other licenses held by the same operator. In other words, value of some combinations of spectrum licenses is higher than the sum of the individual license values. For two licenses A and B this can be summarized with a simple inequality $V_{A+B} > V_A + V_B$, where $V_i$ denotes the value of license in question.

Complementarities usually stem from technological or geographical reasons. For an operator it would be highly beneficial to win rights to adjacent spectrum blocks as that would loosen the technological restrictions when developing the services and allow for higher transfer capacity. On the other hand, especially in markets like the US, the value of spectrum licenses often depends on the licenses held by the same operator in adjacent geographical areas. The license for block A in New York is much more valuable to the operator holding the rights to the same frequencies in New Jersey than to other operators.(Chan et al., 2003)

Recognizing the complementarities is just as important to the auctioneer than it is to the operators because failure to recognize complementarities when dividing the spectrum into smaller blocks could give some operators an unfair advantage in the end market or prevent the auction from reaching an efficient outcome. Analyzing the complementarities also helps the auctioneer to set the reserve prices[3] as accurately as possible. Setting the reserve prices too high could result in unsold spectrum while setting them too low could slow the auction process unnecessarily. Moreover, the possibility of complementarities eliminates the use of some traditional auction mechanisms, such as Vickrey-Clarke-Groves (VCG) mechanism, that rely on the assumption of uniform valuations.[4](Cramton, 2013)

[3]Reserve price is the starting price in the auction. It can also thought to be the government's opportunity cost for selling the spectrum. At prices below reserve price it would be better for government to hold on to the spectrum rights instead of selling them.

[4]VCG mechanism will be discussed in greater detail in Chapter 3.3

## 2.2 Goals of spectrum auction design

In an auction with private sellers the auction design process is usually very straightforward as the only question in seller's mind is 'How can I maximize the revenue?'. However, when the auctioning party is the government, things get more complicated. If the government was only thinking about revenue maximization when allocating the spectrum, the best solution would almost certainly be to sell all of the spectrum rights to the party that offers to pay the highest price. The problem is that this would create a monopoly in the end market for wireless services, increasing consumer prices, driving other operators out of business, increasing unemployment, and likely slowing down the technological advancement. While revenues gained from the auction would be high, it would not be enough to offset these negative externalities. Therefore, instead of taking the revenue maximization approach, the government should aim to maximize the social benefit gained from the spectrum. To achieve this, the auction must be designed with several goals in mind. As Binmore and Klemperer (2002) summarized, the auction design should aim to

- Assign the spectrum efficiently;
- Promote competition;
- Realize the full economic value (subject to other objectives).

It is important to understand that setting the design goals for an auction is not an exact science, and arguments could be made for lengthening or shortening the list, but it should also be kept in mind that the auction mechanism must remain simple, transparent, and fair. Adding a long list of design goals can unnecessarily increase the complexity of the auction design, which can in turn affect the efficiency of the auction in unforeseen ways. In fact, Bazelon (2009) argues that the abundant design goals were the main reason for the poor results in the FCC's auction for 700 MHz licenses in 2008.

### 2.2.1 Efficient allocation of spectrum

Efficient allocation of spectrum should be the number one priority for the government. First of all, the government should ensure that all of the spectrum is put into use. This is an important consideration when choosing the reserve prices, as discussed in Chapter 2.1. If the reserve prices are set too high the government faces the threat of having unsold spectrum at the end of the auction. This is the problem Turkey faced in 2000 when it auctioned off two spectrum lots in sequential auctions. For the second auction the reserve price was the sale price of the first lot. However, the price ended up being too high for any other operator to pay, and the second block was left unsold.

The second aspect of efficient allocation of spectrum is to award the licenses to the operators that have the resources and means to put the spectrum into best possible use. Coincidentally, this often corresponds to firms that value the spectrum most, which further strenghtens the argument for paying less attention to the revenue: if the allocation is efficient, the auction should, by default, raise high revenues.

### 2.2.2 Promotion of competition

The auction design should be such that it allocates the spectrum in a manner that creates a competitive end market. This way the wireless services are available to the consumers at reasonable prices and the competitive environment encourages companies to develop the services further speeding up the technological advancement in the country. This is generally achieved by dividing the spectrum into smaller blocks and limiting the number of blocks any operator can win in the auction. The division of spectrum must be done carefully, as discussed in Chapter 2.1. In Chapter 5 we will see how combinatorial clock auction model helps the government by shifting the allocation problem partly to the bidders.

The auction should also be designed in a way that it increases competition during the auction process, i.e., makes participants to bid competitively. This means that the auction mechanism should promote truthful bidding in order to enable price discovery. This is usually achieved by enforcing certain activity and eligibility rules, which will be discussed in more detail in the following chapters.

### 2.2.3 Revenue considerations

So far this chapter has introduced arguments against favoring revenue maximization in the auction design process, but it does not mean they should be ignored completely. In fact, if the revenue aspect of spectrum allocation was ignored, one could argue that auctions are not needed at all. There are at least two important reasons why it is important to organize an auction that raises as much revenues as possible, as long as it is not prioritized over efficient allocation and competitive end market. First of all, the revenues raised by the government can be used to increase social gains even further. Whether the money collected from the auction is used to improve infrastructure or reduce taxing, it is going to improve the social wellbeing of the consumers. Second of all, when companies have to pay significant sums to acquire the spectrum rights, they are more likely to stand behind their business plans and even encourage them to develop the services faster. (Cramton and Schwartz, 2002; Binmore and Klemperer, 2002)

The most common argument against collecting revenues from spectrum allocation is that the firms' costs will be passed on to the consumers. There is, however, a major flaw in this thinking: once the companies start to offer the wireless services to the consumer, the price they paid for the spectrum price is a sunk cost and should not affect their pricing decisions in any way. (McMillan, 1995)

# 3 Multi-unit auction models

Whenever auctions are mentioned, most people would automatically start thinking about traditional auctions where throughout the auction the price for an item increases as participants place bids exceeding the previous standing high bid. This is the most common auction model and is extensively used in, e.g., art auction and online auctions. Auctions like these can be characterized as ascending price single-item auctions and are often called English auctions.

Alternatively, auction can also have descending prices where the auctioneer announces a price in the beginning of an auction and starts to lower it little by little until one of the participants accepts the price and wins the item. An auction like this is usually called a Dutch auction, for historical reasons.

Auctions can also be classified as either open or closed auctions. In an open auction, like the English auction, the bid information is public to all participants, i.e., every bidder knows the current price at all times. In a closed auction (often referred to as a sealed-bid auction), the participants submit bids to the auctioneer who then reviews all the bids and announces the winner – the bidder who placed the highest bid. The most common sealed-bid auction type is the Vickrey auction that uses second price pricing: The winner of the auction is the bidder who placed the highest bid, but the price they pay for the item is the second highest bid placed in the auction, i.e., the highest losing bid.

Aforementioned auction models are all single-item auctions where only one item is sold at a time. However, in many cases this is not enough but instead the auctioneer wants to sell several items at the same time. Auctions like these are called *multi-unit auctions*, and they are the main focus of this chapter.

Selling multiple items at the same time naturally complicates the auction design because in most cases additional rules are required to define, for example, the bidders' ability to shift bids from one item to another or withdraw bids – something that does not need to be considered or allowed in single-item auctions. For bidders the decision-making process gets more complicated as they will have more options available to them. This chapter covers some of the most common multi-unit auctions and issues related to them while also laying the foundation for better understanding the spectrum auction models that will be discussed in detail in Chapter 4.

## 3.1 Simultaneous ascending auction

Simultaneous ascending auction (SAA) is an extension of English auction but for multiple items that are sold simultaneously. In a SAA an own English auction is set up for each of the items to be sold, and the auctions are ran concurrently. Each auction consists of multiple rounds and they proceed in unison. Typically after each round the auctioneer announces the standing high bids for all items and the bidders currently holding said bids. In the following round the bidders currently not holding the highest bid for any of the items can place new bids that must exceed the current high bid by at least a pre-determined minimum price increment, while current high bidders often have no incentive to place additional bids. Most of the time the auction rules dictate that each bidder must remain active throughout the auction, i.e., place a bid unless they were holding the highest bid at the start of the round, in order to increase transparency and reduce bid sniping opportunities[5].

The auction ends once none of the items receive qualifying bids during a single round. The main advantage over sequential auctions is that in a SAA the bidders have the ability to observe the prices of all items throughout the auction and can therefore bid on the most profitable one(s) each round. In a sequential auction the bidders would have to consider not only their valuation and the current price in the auction, but also the possible outcomes of the following auctions.

Simultaneous ascending auction process is easiest described with a simple example where four homogeneous items are sold among five participants. If the items are truly homogeneous and the bidders understand it, all of them will value all of the items the

[5]Bid sniping refers to the action where bidders hide their preference early in the auction just to jump in at the last minute and place a large bid.

same, and therefore they are indifferent regarding which item they win. In this case the optimal strategy is to always bid on the cheapest available item as long as any item's standing high bid is below their private valuation. Suppose that the starting price for the items is 100, minimum price increment is 10, and private valuation of each participant is as described in Table 1.

Table 1: Bidders' valuation for a single item

| **Bidder** | B1 | B2 | B3 | B4 | B5 |
|---|---|---|---|---|---|
| **Valuation for a single item** | 200 | 200 | 180 | 170 | 150 |

In the first round each bidder places a bid of 100 on one of the items and the outcome could look something like shown in Table 2.

Table 2: Bids placed during the first round of the auction. Bidder(s) in parantheses.

| **Bidder** | Item 1 | Item 2 | Item 3 | Item 4 |
|---|---|---|---|---|
| **Round 1** | 100 (B1) | 100 (B2) | 100 (B3) | 100 (B4, B5) |

Bidders B4 and B5 are the only ones without a standing high bids after the first round and they will therefore increase their bids. Bidders B1 through B3 are currently holding standing high bids and will therefore not make bids on round 2. Let us assume that B4 decides to bid 110 on item 1 while B5 increases her bid to 110 for item 4. The situation after the second round is shown in Table 3.

Table 3: Standing high bids after the second round of the auction.

| **Bidder** | Item 1 | Item 2 | Item 3 | Item 4 |
|---|---|---|---|---|
| **Round 1** | 100 (B1) | 100 (B2) | 100 (B3) | 100 (B4, B5) |
| **Round 2** | 110 (B4) | 100 (B2) | 100 (B3) | 110 (B5) |

Now B1 is the only one without a standing high bid and she will therefore place a bid of 110 on one of the two items currently priced at 100. Other bidders have no reason to make additional bids on round three as they are currently holding the highest bid on an item. If B1 bids on item two, then B2 will face a similar situation in round 4 and will bid 110 on item 3, the only one priced at 100 at that point.

At this point the auction has been going on for four rounds, all the items are priced at 110, and B3 is the only one without a standing high bid.[6] The same process would then repeat over the next four rounds resulting in all items priced at 120 and one bidder without a standing high bid. This would go on until all the prices eventually reach 150, the lowest private valuation, after 20 rounds. At this point the allocation could look something like shown in Table 4. Auction ends once B5 is outbid for the final time.[7]

Table 4: Auction after 20 rounds.

| **Bidder** | Item 1 | Item 2 | Item 3 | Item 4 |
|---|---|---|---|---|
| **Round 1** | 100 (B1) | 100 (B2) | 100 (B3) | 100 (B4, B5) |
| **Round 2** | 110 (B4) | 100 (B2) | 100 (B3) | 110 (B5) |
| | | . | | |
| | | . | | |
| | | . | | |
| **Round 20** | 150 (B1) | 150 (B2) | 150 (B3) | 150 (B4,B5) |

If a single bidder is allowed to win multiple items, the decision-making in a SAA gets a little bit more complicated as the bidders must determine the most profitable package each round, i.e., consider the marginal value of additional items. From the seller's point of view the SAA model is a relatively safe auction model but can require plenty of resources as noticed in the previous example. SAAs and its applications are commonly used in the sale of spectrum rights and the intricacies of the model will be discussed in detail in Chapter 4.

[6]During round 4 B3, the bidder holding the bid of 100 on item 3, could have predicted B2's intentions and therefore increased her own bid. However, if B2's valuation was below 110 and she was therefore not going to make an additional bid, B3 would have ended up overpaying.

[7]This could happen already in round 21 if B4 increases her bid on item 4, and will at latest happen after round 24 when all items reach the price of 160.

## 3.2 Clock auction

Clock auction is a type of ascending price multiround auction where the auctioneer announces prices for the items to be sold and participants submit their demands at current price. The simplest version of the clock auction is one where each bidder can win a maximum of one item. In this case the auctioneer announces the price and participants signal either yes or no to indicate whether they are willing to pay that price or not. Once the number of bidders signaling yes equals the number of items to be sold, the auction ends and the items are allocated to the bidders signaling yes in the last round.

If winning multiple items is allowed, the bidders signal the amount of items they are willing to buy at the current price. After each round the auctioneer sums the demands and if the aggregate demand exceeds the supply, the auction moves on to the next round with increased prices, and participants submit new demands. Once the demand drops to the level of supply or below it, demand is said to be *market clearing*, auction ends, and the items are allocated according to the participants' demands.

The most common clock auction format uses *uniform pricing*, where all items are sold at the market clearing price. However, another option is to use a clinching rule originally proposed by Ausubel (2004). In this case some items may be clinched before the final round. This happens when the total demand is above supply but the total demand without a specific bidder's demand is below supply. In other words, if $D_{TOT} > S$ and $D_{TOT} - D_{B_i} < S$ then bidder $B_i$ would clinch $S - (D_{TOT} - D_{B_i})$ items. This is because without bidder $B_i$ these items would be left unsold and therefore they are allocated to bidder $B_i$ at the current price.

The following example illustrates the clock auction process with clinching rule in more detail. Consider an auction where 30 homogenous items are sold among five participants A, B, C, D, and E. The starting price per item is 1 and it is increased in increments of 0.10. Table 5 shows the evolution of demands throughout the auction. The auction ends after round 7 when demand is equal to the supply of 30.

Table 5: Clock auction outcome for ten homogenous items.

| **Round** | **Price** | **A** | **B** | **C** | **D** | **E** | **Post-round actions** |
|---|---|---|---|---|---|---|---|
| **1** | 1 | 15 | 12 | 12 | 10 | 8 | Price increased |
| **2** | 1.1 | 12 | 10 | 12 | 10 | 8 | Price increased |
| **3** | 1.2 | 12 | 8 | 10 | 8 | 5 | Price increased |
| **4** | 1.3 | 12 | 8 | 8 | 8 | 5 | A clinches 1 item |
| **5** | 1.4 | 10 | 7 | 7 | 7 | 5 | A clinches 3; B,C, and D clinch 1 |
| **6** | 1.5 | 10 | 6 | 6 | 6 | 5 | A clinches 3; B,C, and D clinch 2 |
| **7** | 1.6 | 10 | 6 | 6 | 5 | 3 | Auction ends |

Table 6 shows the final allocation and prices each participant pays, as well as the average price per item. Bidder A who submitted the highest demand throughout the auction was rewarded with the lowest average price per item and therefore the example shows how the clinching rule can be used to encourage truthful bidding. Without the clinching rule the outcome with the same bidding behavior would be more favorable to the auctioneer as all the items would be sold at a price of 1.6, which is approximately 5 % higher than the average price with the clinching rule. However, this might encourage participants to collude, which could result in lower-than-expected revenues for the seller. Collusion, especially in the form of demand reduction, will be discussed in more detail later in this chapter.

Table 6: Summary of clock auction results.

| **Price** | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|
| **1.3** | 1 | 0 | 0 | 0 | 0 |
| **1.4** | 3 | 1 | 1 | 1 | 0 |
| **1.5** | 3 | 2 | 2 | 2 | 0 |
| **1.6** | 3 | 3 | 3 | 2 | 3 |
| **Total** | **10** | **6** | **6** | **5** | **3** |
| **Average price** | **1.48** | **1.53** | **1.53** | **1.52** | **1.6** |

The power of clock auction model in allocation of homogenous items is apparent: Using a simultaneous ascending auction model, the sale of 30 items would require 30 individual auctions, whereas the use of clock auction model allowed the same allocation to be reached with one auction and only seven rounds. Similar result can be noticed by revisiting the example used to describe the SAA process where four homogenous items were sold among five bidders. If the same items were sold in a clock auction the price for the items would increase by 10 each round resulting in the auction ending after only six rounds, compared to 20+ rounds SAA model required.

These two examples show how the use of clock auction model can significantly reduce the resources required to sell the items as well as simplify the process for bidders compared to simultaneous ascending auction model. Clock auctions are widely used in, e.g., sale of electricity and emission permits. Clock auctions have one big inherent problem: it is perfectly possible that the demand drops from above supply to below supply during one round, leaving some of the items unsold. This feature severely limits the clock auction model's suitability for spectrum auctions. However, clock auction model is one of the cornerstones of the combinatorial clock auction model, which is the focus of Chapter 5.

## 3.3 Vickrey-Clarke-Groves auction

Vickrey-Clarke-Groves (VCG) auction is a multi-unit extension of the Vickrey auction, originally proposed by William Vickrey in (1961). Vickrey discussed the strengths of the second-price sealed-bid auction in selling individual items and showed that truthful bidding is the dominant strategy for the bidders. It is easy to see why this is the case. Let us consider a situation where two bidders A and B are participating in an auction for an item, and A's private valuation for the item is 6. If A bids above her valuation $b_A = 6 + \alpha$, where $\alpha > 0$ she enables a situation to occur where B's bid is $6 + \beta$, where $0 < \beta < \alpha$. This would mean that A wins the auction and pays B's bid, which would lead to a profit of $-\beta$ and A would have been better off not bidding at all and ending up with a profit of zero. On the other hand, if A bids below their valuation $b_A = 6 - \alpha$ then it is possible that B bids $6 - \beta$ and A would have made a profit of $\beta$ by bidding

$b_A = 6$. Only by bidding her valuation can A guarantee that she never must pay above their valuation and never misses out on potential profit. An auction like this where truthful bidding is optimal is said to be *incentive compatible.*
Later Clarke (1971) and Groves (1973) showed that the model extends well to multi-unit auctions and that the dominant strategy is still to bid truthfully. The multi-unit model is often referred to as Vickrey-Clarge-Groves mechanism, or VCG for short. The VCG mechanism is studied extensively in the literature and detailed analyses can be found in, e.g., (Milgrom, 2004c) and (Mochón and Sáez, 2015). The results show that in a VCG auction, as in a Vickrey auction, bidding truthfully is a dominant strategy for the bidders. Moreover, if the bidders do bid truthfully the auction leads to an efficient allocation of items.

The following example demonstrates the use of VCG mechanism in an auction with four bidders B1, B2, B3, and B4 for two items A and B. The bidding behavior is presented in Table 7.

Table 7: VCG auction example with four bidders and two items.

| **Item** | **B1** | **B2** | **B3** | **B4** |
|---|---|---|---|---|
| **A** | 6 | 0 | 2 | 3 |
| **B** | 0 | 6 | 4 | 2 |

Having placed the highest bids, bidder B1 wins the item A and bidder B2 wins the item B. The prices they pay for the items are, respectively, 3 and 4. Figure 3 shows the outcome of the auction graphically where each line represents an individual bid. The shaded area is called the core, which is defined, as Cramton (2013) puts it:
"*The core is defined as set of payments that support the efficient assignment in the sense that there does not exist an alternative coalition of bidders that has collectively offered the seller more.*"

All points within the core present an efficient solution to the pricing problem and Vicrey-solution lies in the south-west corner of the core, which means that it is the

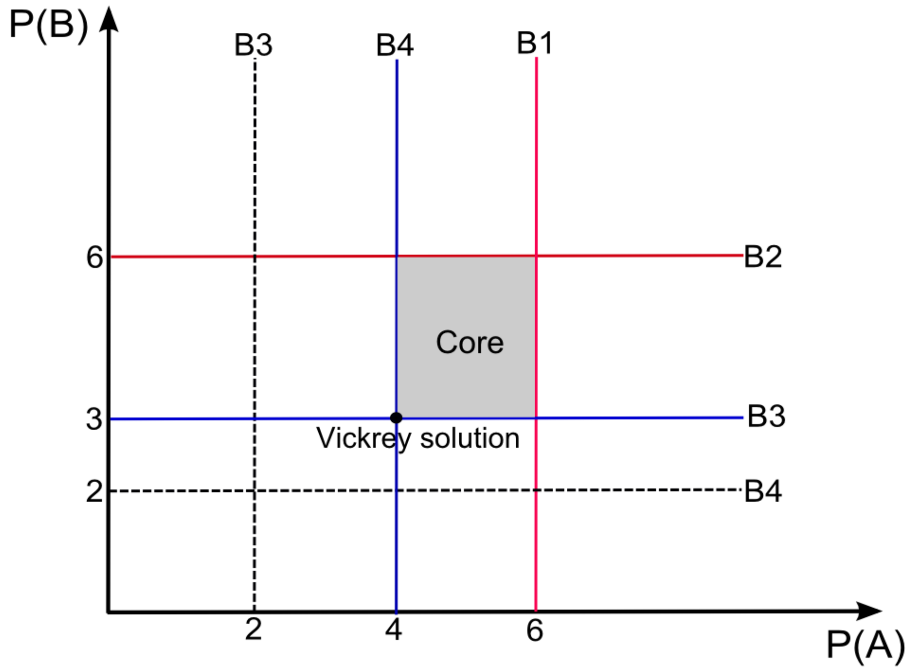


Figure 3: Vickrey solution for multi-unit auction.

bidder-optimal solution within the core. In other words, Vickrey prices are the lowest prices that still guarantee an efficient allocation. It is worth noting that the number of items or bidders does not complicate the VCG auction because only the highest and second highest bids are important for finding the efficient allocation of items (just like in Figure 3 the bids illustrated with dashed lines play no role in finding the winners and payments).

While VCG mechanism is resource-friendly and results in an efficient outcome, it does increase variation in revenue for the seller, which might make risk-averse sellers hesitant to use it. To illustrate this, consider the previous example but now the bidder B1 is the only one interested in item A and therefore the item receives only one bid in the auction. This would result in the selling price for A to be zero, while with a simultaneous ascending auction model the price would be at least the reserve price for the item.

A version of VCG mechanism could also be applied in the example situation discussed before where five homogenous items were auctioned among four bidders. In this case instead of second price auction, a $n$th price pricing is used, where $n$ equals the number of items plus one. Since all the items are substitutes, the auctioneer can ask bidders to submit bids indicating their willingness-to-pay for a single item and then allocate the items for four bidders with the highest willingness-to-pay. The final price would be the highest losing bid, i.e., the fifth highest bid. A model like this can be very effective as the auction only requires one round of bidding and the optimal strategy is to bid truthfully. However, if the items are not substitutes as is often the case in spectrum auctions, the model runs into several problems. The shortcoming of the VCG model in spectrum auctions will be discussed in detail in Chapter 5.

## 3.4 Typical issues in multi-unit auctions

This section covers some of the most common issues that can lead to an inefficient auction outcome in multi-unit auctions. These issues are an important consideration for the seller when deciding which auction mechanism to use. Empirical evidence can be found of many of these problems arising in real life spectrum auctions. These will be discussed in more detail in Chapter 4.3 while this chapter concentrates on the theoretical aspects of the issues.

#### Demand reduction

Demand reduction is a form of collusion that refers to bidders purposely reducing their demand in an auction with the intention of making the auction end early and at low prices. In a worst-case scenario (from seller's point of view) the participants reach a silent agreement regarding the allocation of items in the opening round and the items are sold at reserve prices. If bidders understand that bidding aggressively is going to not only increase the prices for other participants but for themselves as well, demand reduction can take place even if bidders are not explicitly colluding. In an auction where bidders have declining marginal values, the incentives for reducing demand early on can be particularly strong. Some auction models, such as uniform price clock auction, are especially vulnerable to demand reduction as there may exist low-revenue equilibria encouraging bidders to collude. Demand reduction and low-revenue equilibria are studied extensively by, e.g., Ausubel and Schwartz (1999), Engelbrecht-Wiggans (2005), and Akaichi et. al. (2014).

**Signaling**

Signaling refers to bidders using different means to communicate their preferences and interests to other participants throughout the auction in order to discourage them from bidding on the items they are interested in themselves. Signaling can be especially beneficial for bidders in simultaneous ascending auctions where each item is sold in an individual auction. Common signaling tactics include, e.g., jump bids, retaliatory bids, bid withdrawals, and use of trailing digits.

Jump bidding refers to increasing a current highest bid by more than the minimum bid increment. This way bidders can message others that they are extremely interested in the item in question and that the others should stay way. Retaliatory bidding refers to one bidder punishing another for bidding on the ”wrong” item. For example, consider a situation where bidder A has the standing highest bid on item 1 and bidder B has the standing highest bid on item 2. Bidder A now places a bid on item 2 outbidding bidder B. Bidder B can either increase their bid on item 1 or retaliate by outbidding bidder A on item 1, regardless of their true preferences.

Bid withdrawals are not as common and require specific auction rules where withdrawing standing high bids are allowed. Under these condition bidders can place the highest bid on an item and immediately withdraw it and that way signal that they are not truly interested in that specific item. Finally, the use of trailing digits refers to tailoring the bid amounts in a way that the final digits in a bid send a message indicating their preferences.

As it turns out, bid signaling has been used extensively in spectrum auctions and empirical evidence of all aforementioned signaling tactics can be found from the literature, and will be discussed in greater detail in Chapter 4.3.

**Winner's curse**

Winner's curse is a phenomenon where the true value of the item is uncertain and overestimation leads to overbidding. In this case the winner of the auction is the party that has made the largest mistakes when estimating the value of the item. Winner's curse can occur especially in first-price auction where the payment is equal to the winning bid. Even though winner's curse may lead to high revenues for the seller, the problem is that when the bidders are aware of the possible winner's curse, i.e., there is significant uncertainty in valuations, they are likely to bid more cautiously in the auction and that way decrease the revenues for the seller. Therefore it is in the interest of all parties that winner's curse is eliminated from the auction. (Klemperer, 2002)

# 4 Auction models used in spectrum allocation

This chapter shifts the discussion to spectrum auction environment by presenting the most common auction models used in spectrum allocation as well as discussing the typical issues that have been faced in real-world spectrum auctions over the past decades.

## 4.1 Simultaneous multiround auction

Ever since the introduction of auctions in spectrum allocation by the FCC in 1994, simultaneous multiround auction (SMRA) has been by far the most popular auction model for spectrum allocation, and vast majority of spectrum auctions all over the world have used some modification of it. SMRA is essentially a simultenous ascending auction (described in Chapter 3.1), where each spectrum block is sold in its own auction, and all auctions are ran simultaneously. The opening bids are determined by the reserve prices set by the government before the auction. There is no predetermined number of rounds but rather the auction continues as long as there are active bidders in any of the auctions. The auctions use sealed bids and after each round participants are informed about the highest bid on each item, and usually the bidder identity as well.[8]

### 4.1.1 Auction rules

Over the past decades governments have incorporated several additional rules to the SMRAs, the most important one being the *activity rule*. Activity rules are used to ensure that participants must be active throughout the auction, i.e., they can not sit back early on and jump in at the later stages of the auction. The main purpose of the activity rules is to increase the openness of the auction by encouraging participants to reveal their true preferences and therefore enable price discovery. The most common activity rule in SMRAs is a simple monotonicity rule that states that if a bidder cur-

[8]The level of transparency may vary a lot from auction to auction. In some cases only the bid amounts are released to the participants between the rounds but not bidder identities. Similarly, after the auction, in some cases full bidding activity throughout the auction is released to the public (e.g., German 4G auction in 2010) while in some other cases only the winners and the winning bid amounts are released (e.g., Finnish spectrum auctions).

rently not holding the highest bid for any of the licenses fails to submit a qualifying bid during a round, loses their eligibility to bid on any of the licenses in the further rounds. Usually due to the high stakes of the spectrum auctions the auctioneer grants bidders a certain amount of waivers that allow bidders to skip a round in order to reconsider their valuations, or negotiate with their financiers. Another common variation of the activity rule is a quantity-based rule, which is useful in auctions where bidding on multiple licenses is allowed. In this case, bidders are not allowed to increase the number of licenses they bid on during the auction. For example, if a bidder bids on three licenses in the first round, they can bid on maximum of three licenses in the future rounds. In other words, bids must be consistent with a downward sloping demand curve. (Ausubel et al., 2006)

Bid withdrawals may be allowed in order for participants to be able to re-evaluate their preferences throughout the auction. However, allowing for withdrawals has the drawback of possibly increasing the length of the auction or open up the possibility for predatory activities, and therefore withdrawals often result in a penalty. After a standing high bid is withdrawn, the auctioneer becomes the standing high bidder and the reserve price for the item is the second highest submitted bid.
While uniform pricing (i.e., all items are eventually sold at the same price) may be applied in SMRAs, pay-your-bid rule is generally more preferable because it gives bidders less incentive to strategically reduce demand early in the auction. It is worth noting though, that if items are pure substitutes, pay-your-bid pricing is likely to end with all items selling for approximately the same price since bidders will always bid on the cheapest item. Minimum bid increment is usually set at 5-10 % to control the length of the auction. (Cramton et al., 2006)

SMRAs have proven to work well in many situations but several issues still persist. More specifically, SMRAs have worked well in auctions where there are no technological or geographical complementarities, and all bidders' valuations for the licenses are homogeneous. However, when these conditions are not met, especially when complementarities are present, issues such as exposure risk may become a reality and SMRA often fails to achieve an efficient allocation of the spectrum. (Chan et al., 2003; Bichler and Goeree, 2017; Milgrom, 2004a)

## 4.2 Ascending clock auction and variants

Due to relatively high likelihood of inefficient outcomes, ascending clock auctions have not been a popular choice for spectrum allocation in the past. For this reason it is difficult to discuss the auction process and rules the same way that was done for SMRA. Instead, some of the auctions that have implemented some variant of ascending clock auction model will be discussed here.

In 1999 the German 3G licenses were sold using a sequential ascending clock auction model where during each round one participant places a bid which is then used as a reserve price for the following round where another bidder places their bid. The auction ends when participants accept the allocation suggested by the current highest bids. Accepting the allocation here means that any of the bidders do not outbid the standing high bid on any of the licenses. Once every participant has passed on the option to place a qualifying bid, i.e., accepted the allocation at current prices, the auction ends. The German auction had only two participants and ten homogeneous licenses for sale, which made the auction relatively simple. Uniform pricing was used and the minimum bid increment was set at 10 %.(Grimm et al., 2003)
A simultaneous ascending clock auction was applied in the first Nigerian spectrum auction in 2001 where three available licenses were auctioned among five participants. The setting was especially favorable for a simultaneous clock auction as the licenses were considered homogenous and any participant could win a maximum of one license. To address the possibility of having the demand drop from above supply to below supply during a single round, leaving unallocated licenses, the Anglo-Dutch variant of a clock auction was used. The Anglo-Dutch auction was first proposed by Klemperer (2002)

and it was considered an option for the UK spectrum auction in 2000. The model is a two-stage process which consists of a clock phase and a single-round sealed-bid $n$:th price auction. In Nigeria's case this would have meant that if the demand had dropped from, e.g., 4 to 2 over a single round then the unallocated item would have been sold using a third-price sealed-bid auction among the bidders who had not yet secured a license in the clock phase. Furthermore, had the number of participants exceeded the number of licenses by only one, a fourth-price sealed-bid auction would have been used instead of a clock auction. However, since neither of these conditions were met in the auction, the licenses were effectively allocated using a simultaneous ascending clock auction. During each round the auctioneer announced the prices for the licenses and the bidders submitted a YES or NO answer indicating whether they accept the price or not. The bidders were also granted three waivers that could be used during any round of the auction. The maximum round-to-round price increase was set at 10 % but since the participants were not allowed to consult their financiers during the auction, an additional rule stated that the price per license could only increase by 50 % during any day. If this condition is met, the auction would end for the day giving participants an opportunity to discuss strategy with their financial backers. The auction lasted for three days.(Lee, 2003)

Even though ascending clock auction model has been overshadowed by the SMRA model in spectrum allocation, it can be advantageous in a competitive setting with several homogeneous spectrum licenses. In this case ascending clock auction is likely to produce the same allocation as SMRA but in much fewer rounds, as discussed in Chapter 3.2. More importantly, ascending clock auction is a vital part of the combinatorial clock auction (CCA) model which is widely considered the most prominent allocation method for spectrum licenses. CCA model will be covered in detail in Chapter 5.

## 4.3 Issues faced in spectrum auctions

Ever since the adoption of auctions in spectrum allocation, governments have faced several issues related to them. Some of the issues are specific to certain auction types while others are more universal problems with multi-object auctions. This chapter covers the most serious problems related to spectrum auctions. Many of these issues have been successfully eliminated by enforcing strict rules, but others still pose a considerable risk when organizing spectrum auctions.

### Demand reduction

Demand reduction is one of the main concerns in all multi-unit auctions. If several identical products are sold at the same time, there is a clear incentive for the participants to stop bidding early and therefore keep the prices low for all the items, as discussed in Chapter 3.4. One of the most obvious cases of demand reduction is discussed by Grimm et. al. (2003). In 1999 the German 3G auction, briefly mentioned in the previous chapter, was implemented using a sequential clock auction model and the auction rules stated that each new bid should exceed the previous highest bid by at least 10 %. Total of 10 frequency blocks were sold in the auction and there were only two major players participating in the auction, Mannesman and T-Mobile. In the first round Mannesman bid DM 18.18 million for blocks 1-5 and DM 20 million for blocks 6-10. By doing so Mannesman was suggesting a 50-50 division of spectrum: by increasing Mannesman bid on blocks 1-5 by the minimum of 10 %, and not challenging its bid on blocks 6-10, both companies would win five blocks at a price of DM 20 million. T-mobile understood this, bid accordingly, and the auction ended after only two rounds resulting very low revenues for the government.

Bichler et. al. (2017) analyze the bidding behavior in the German 4G auction in 2015 and find clear evidence of participants trying to find a collusive agreement using different kind of signaling strategies throughout the auction. While in this particular action the participants did not find a collusive agreement regarding the allocation of the licenses and the auction ended up being relatively successful, the auction rules did enable an opportunity for bidders to collude.

**Signaling**

Bid signaling refers to the action of bidders where they bid according to certain rules in order to inform their competitors of their strategies. One of the most common ways to do so is the use of *retaliatory bids*. Retaliatory bids are one bidder's way of punishing the competitors who are bidding on a license they want to acquire. Cramton and Ockenfels (2017) find evidence of retaliatory bidding behavior in the German 4G auction held in 2010, while Cramton and Schwartz (2000) describe the use of retaliatory bids in an FCC auction for personal communication services in Texas in 1996-1997. What makes the latter case interesting is that the retaliatory bids were combined with the use of trailing digits, another signaling tactic. Two participants, Mercury PCS and High Plains Wireless, were battling to win the license for Lubbock, Texas. On round 121 of the auction Mercury PCS placed a highest bid on another license High Plains Wireless was the standing bidder on. This was Mercury's first bid for the specific license, and the bid amount ended with digits "264", which was the area code for Lubbock, Texas. This was Mercury PCS's way of telling its competitor that if they do not back off from Lubbock license, they will be punished elsewhere and will have to pay a higher price for the other licenses. The use of trailing digits has since been successfully prevented by implementing rounding requirements for the bid amounts.

Other means of signaling include actions such as jump bids (increasing a bid by more than the minimum amount (Bajari and Yeo, 2009)) and bid withdrawals (bidding high on a certain license and then immediately withdrawing the bid (Cramton and Schwartz, 2000)).

Many of these methods can be eliminated by enforcing strict rules in the auction, but bid signaling in general can be difficult to eliminate without decreasing the openess and transparency of the auction process.

**Exposure risk**

Exposure risk arises when a bidder's valuation for a bundle is higher than the sum of individual products' stand-alone values. This can happen when there are complementarities between different products and package bidding is not allowed. Consider a VCG auction where two identical spectrum blocks P1 and P2 are being sold. Bidder A's valuations $V_i$ for P1, P2, and the combination P1+P2 are $V_{P1} = 4$, $V_{P2} = 4$, and $V_{P1+P2} = 12$. As discussed previously, in a VCG auction the dominant strategy is to bid truthfully and therefore if any bidder could win a maximum of one product, it would be optimal for A to bid 4 for both products. However, if winning multiple items is allowed, it becomes optimal for A to increase her bids for both blocks, therefore bidding more than either block's stand-alone value. When doing so, A runs the risk of winning only one block and paying too high of a price for it. This is referred to as exposure risk and if it happens, the allocation is not efficient because the full value of the spectrum is not realized. Moreover, in a competitive auction a risk-averse bidder is likely to bid maximum of 4 for each block, therefore restricting herself from bidding according to her true valuations.(Chan et al., 2003; Cramton et al., 2006) In SMRAs exposure risk is often tackled by allowing bidders to withdraw standing high bids or shift their bids from one block to another, but this increases the bidders' ability to use signaling tactics by, e.g., exercising predatory bidding. Another option to limit exposure risk is to allow re-sale of spectrum blocks after the auction but the downside is that this may attract insincere bidders who are looking to win licenses with only re-sale in mind, which can easily lead to inefficient allocation. Cramton and Ockenfels (2017) analyze the bidding behavior in the German spectrum auction in 2010 and speculate that this might have been E-Plus's objective in the auction. E-Plus reduced demand early in the auction possibly hoping that it might incentivize larger operators to follow suit and therefore create a low-revenue outcome.

**Unallocated spectrum**

One of the worst possible outcomes in a spectrum auction is one where significant part of the spectrum is left unallocated. The biggest issue with unallocated spectrum is that it can be difficult for the government to sell the unallocated licenses after the auction has ended without hurting its credibility in the future auctions. If, for example, unallocated licenses were solds after the auction for a lower price than in the auction, the bidders in future auction may not take the auction seriously knowing that if all of the licenses are not sold they might have a real chance of acquiring those licenses for a cheaper price after the auction.(Klemperer, 2002) Moreover, if all of the spectrum is not allocated the auction outcome is not likely to be efficient either. Therefore it is important for the auctioneer to design the process in a way that ensures the full allocation of the spectrum (or as close to it as possible). Fortunately in majority of spectrum auction worldwide there has been no significant portion of the spectrum left unallocated but this was the case in, e.g., Turkey (2000). In the first Nigerian spectrum auction, described in the previous chapter, one of the winners ended up defaulting on their payment transferring the spectrum right for that specific license back to the government. While payment default issues may sometimes be difficult to foresee they can be minimized by ensuring that the participants are serious about the auction. In the developed world payment defaults are far less likely due to strong institutions and functioning capital markets, and a simple entry fee is often enough to ensure seriousness of bidders. In the developing world the auctioneer often requires the participants to make sizable deposits prior to auction. These deposits will count towards the payments and therefore act mostly as a proof of financing, although they can also be used to enforce auction rules by defining the deposit forfeited in case the rules are broken.(Lee, 2003)

# 5 Combinatorial clock auction model in spectrum auctions

Over the past decade the most prominent auction model for spectrum auctions has been the combinatorial clock auction (CCA) model, which combines package bidding with dynamic price discovery. The main fundamental difference between combinatorial auction model and SMRA is that the former allows bidders to bid on arbitrary bundles of items, rather than on single items or pre-determined packages. This feature limits or completely eliminates the exposure risk, which has been an issue with SMRA and other models in the past. Elimination of exposure risk has become increasingly important in modern spectrum auctions where it is common to sell licenses for several frequency ranges in one auction, which often creates more complementarities across licenses as well as increases the number of potential packages bidders might be interested in.

The use of CCA model in spectrum auctions was first introduced by Ausubel, Cramton, and Milgrom in 2006, and it was adopted quickly after that by the UK communications regulator Ofcom in 2008. Since then, the popularity of CCA model in spectrum auctions has increased rapidly and it has replaced SMRA as the most popular auction model in spectrum allocation. So far CCA model has been successfully applied in spectrum auctions in, e.g., Denmark (2010), The Netherlands (2012), and Canada (2014 and 2015).(Mochon and Saez, 2017)

## 5.1 Auction process

Combinatorial clock auction is a two-stage process: The first stage is called *the clock round* and the second stage is usually referred to as *supplementary round*.[9] During the clock round the prices are increased step by step by the seller and bidders respond by submitting their demands for packages they are interested in at each price level. In the supplementary round the bidders can improve their bids from the clock round as well as submit bids for new packages they are interested in. After the supplementary round

[9] Clock round is often also referred to as allocation stage while common alternative names for the supplementary round are assignment stage and proxy phase.

a winner-determination algorithm is used to solve for the optimal allocation of licenses. While the auction process is fairly complex, strategically the auction can become simple as the auction rules are structured in a way that truthful bidding is strategically optimal regardless of competitors' strategies.

### 5.1.1 Clock round

Clock round works like a traditional clock auction where the auctioneer raises the prices step by step and in each round the participants express their demand for spectrum at the current prices. Clock auction is similar to a simultaneous ascending auction used by the FCC, but it has the advantage of being faster when the items are substitutes, as discussed in Chapter 3.2. In the FCC auctions each item has a price associated with it and bidders can bid on any of the items. If the number of items is large and there are many bidders, it can take a long time until the price of each item has increased enough that there is no further bidding. However, when selling a set of homogenous items in a clock auction, the process becomes much simpler. Bidders simply must determine how many items they demand at the current price level and submit their bids to the auctioneer. Ausubel and Cramton (2004) show that bidding truthfully is an optimal strategy in clock auctions and leads to a competitive allocation, which makes the clock auction attractive from seller's point of view. The downside is that clock auctions are especially vulnerable to collusion as there may exist low-revenue equilibria that makes it profitable for bidders to reduce their demand early in the auction to enable low market-clearing prices, as discussed in Chapter 3.4.

In modern spectrum auctions the items are rarely homogenous but significant complementarities across individual items may be present. This complicates the auction design but does not rule out the use of clock auctions. In this case each item has a price associated with it and the prices move individually.[10] If the demand for a specific item is above supply, the item is said to be *oversubscribed*. Likewise, if the demand

[10] There may exist sets of items that are assigned a common price but for the sake if simplicity this possibility is ignored here.

is below supply, the item is said to be *undersubscribed*. At the end of each round the auctioneer raises the price for all oversubscribed items while the prices for other items remain unchanged. This makes the auction somewhat more complicated for the participants. Now the bidders must not only decide *how many* items to demand, but also *which* items to demand. Furthermore, it is possible that for a single bidder there exists multiple optimal choices during one round and in that case the bidder must decide between them as the auction rules generally prevent bidders from placing multiple bids during one round. While this might make it more difficult for the bidders to find the optimal strategy, clock auctions are efficient in eliminating the exposure risk, which is an issue with, e.g., SMRAs.

One inherent problem with clock auctions is that it is easily possible that the auction ends with some items being undersubscribed. For example, consider an auction with ten homogenous items. The demand in round N-1 is $D_{N-1} = 11$, which means that the auctioneer raises the price for round N. Suppose that as a result the demand drops to 9 and the auction ends. Usually a situation like this is prevented by requiring bidders to submit additional information to the auctioneer that helps to determine how to allocate the undersubscribed items at the end of the auction. For example, Ausubel and Cramton (2004) suggest that the auctioneer could require the participants to submit bids also at intermediate prices (prices between $P_{N-1}$ and $P_N$), which could then be used to determine which bidder has the highest marginal valuation for the unsold items.(Milgrom, 2004b)

With heterogenous items, however, things get more complicated. Let us consider a hypothetical situation where six heterogenous items {A, B, C, D, E, F} are sold and the auction has four participants {B1, B2, B3, B4}. The bidding behavior is illustrated in Table 8 where each row shows the bidding behavior of each bidder throughout the auction. For example, in round 1 bidder B2's demand is a set {A, B}.

Table 8: Clock auction with heterogenous items.[11]

| Round | B1 | B2 | B3 | B4 | Post-round actions |
|---|---|---|---|---|---|
| **1** | A | AB | CD | F | Price for A increased |
| **2** | A | EF | CD | F | Price for F increased |
| **3** | A | EF | CD | - | Auction ends |

After round 2 the price for item F is increased causing bidder B4 to drop out of the auction. As a result there are no oversubscribed items and the auction ends. Item B is undersubscribed and is not sold to anyone. Throughout the auction bidder B2 is the only bidder that has placed a bid on item B, but given that she won the package {E, F}, it is unlikely that her marginal valuation for item B is above the reserve price so previous bidding behavior can not be used to allocate the unsold item.

This is where CCA shows its strength. Having unsold items at the end of the auction is not an issue because the auction does not end with the clock round, but just moves on to the second stage where undersubscribed items are sold. The purpose of the clock round is to reveal the bidders' preferences and set the reserve prices for the supplementary round. Once the market clearing demand is reached, the auction moves on to the second stage.

### 5.1.2 Supplementary round

Once the clock round is completed, the auction moves on to the supplementary round with all clock round's winners. In the supplementary round the bidders can place additional bids on other packages they wish to acquire, or increase their standing high bid on the package they won in the clock round. Supplementary round is essentially a single-round auction with sealed bids and modified Vickrey pricing.[12] In a sealed-bid auction all bidders submit their bids to the auctioneer at the same time and have only their private information to work with when determining the optimal bid amounts. This makes sealed-bid auctions more opaque than open auctions, but at the same time

[11] It is easy to see how the example is not very realistic but it still illustrates the problem that may arise in a clock auction with heterogenous items.

[12] The pricing rules of the auction will be covered in greater detail in Chapter 5.2.2.

sealed-bid nature of the auction reduces the threat of collusion.(Ausubel and Baranov, 2014; Athey et al., 2011; Gupta, 2002)

Once the supplementary round ends, all bids, including the those from the clock round, are entered into a winner determination problem and the value-maximizing allocation of the spectrum is solved.

## 5.2 Auction rules

Combinatorial clock auctions, just like any other spectrum auction model, require carefully planned rules in order to eliminate the issues described in Chapter 4.3 and fulfill the auction goals discussed in Chapter 2.2. Many of the rules, such as the limit on how many licenses a single bidder can obtain or the conditions that apply to the resale of licenses, are relatively straightforward. However, there are two types of rules that need to be designed with extreme caution in order to ensure a successful auction: *The activity rules* and the *pricing rules.*

### 5.2.1 Activity rules

Activity rules are a key element in all multi-item auctions but play even a more vital role in spectrum auctions due to the nature of the auction. The most straightforward activity rule simply limits the number of rounds bidders are allowed to be inactive on (not place qualifying bids), and once that number is reached, the next round of inactivity means that the participant is not eligible to bid on any of the following rounds. A rule like that can be viable in certain cases, e.g., if each participant is allowed to win only one block and all blocks are substitutes, like in the Nigerian spectrum auction described in Chapter 4.2. In an auction where bidders are bidding for multiple spectrum blocks, this kind of activity rule would not be effective as participants could simply place bids on very low number of frequency blocks in the early rounds, not revealing their true intentions but still remaining active in the auction.

Another common, slightly more effective activity rule is a quantity-based rule, which restricts the participants from increasing the number of blocks they bid on throughout the auction, i.e., if some participant bids on three blocks in round one, she can ever only bid on maximum of three blocks in the future rounds. A variation of this is an eligibility-point rule, which is based on the same idea but bidders are allowed to bid based on the amount of eligibility points they hold, which depends on their bidding behavior in the previous round. The biggest advantage of these kind of rules is the simplicity, but there are still major issues if there exists complementarities across products. If all blocks are substitutes, participants would simply bid on the blocks with the lowest current prices and reduce their demand if the prices increased too much. However, with complementary blocks, participants would still be incentivized to bid on blocks with the lowest current prices, regardless of whether they are truly interested in those blocks or not. In other words, the participants are not encouraged to bid truthfully. Bidding just to maintain eligibility is often referred to as *parking*.(Mochon and Saez, 2017) Therefore, a more complicated rule is required for CCA model and one based on *revealed preference* was proposed by Ausubel et. al. (2006).

**Revealed preference activity rule**

Revealed preference activity rule states that the bidders can change their bid from one package to another if the package they are shifting to is relatively cheaper. Bidders are allowed to increase the package size, but given that the new package must be relatively cheaper, issues such as bid sniping or parking are not as big of a problem. The rule is designed in a manner that encourages bidders to bid truthfully, i.e., bid on the profit-maximizing package, throughout the clock round in order to maintain as much flexibility as possible for the supplementary round.(Ausubel and Baranov, 2017)

The revealed preference activity rule is best illustrated with an example. The following example is an extension of the one used by Levin and Skrzypacz (2016). Let us consider an auction with two bidders A and B who are competing in an auction for 10 seemingly identical frequency blocks with reserve price per block set at 10. Suppose that Bidder A's valuation for $x \in [0, 10]$ blocks is

$$V_A(x) = 20x - \frac{1}{2}x^2 \tag{1}$$

and therefore her marginal value is

$$V_A'(x) = 20 - x. \tag{2}$$

A's demand function is thus

$$x_A(p) = 20 - p \tag{3}$$

and her inverse demand function is

$$p_A(x) = 20 - x. \tag{4}$$

Bidder A's bid amounts follow the rule

$$b_A(x) = x \cdot p_A(x) = 20x - x^2 \tag{5}$$

In the first round A's demand is 10 blocks and her bid $b_1 = 100$. Meanwhile, according to Equation 1, A values 10 blocks at 150 and therefore her consumer surplus, or profit, is 50. As the auctioneer increases the price per block, A's consumer surplus for each package changes according to Table 9. Bolded values indicate the profit-maximizing package for A at different price levels.

Table 9: Bidder A's consumer surplus for different packages at different price levels.

| **Price** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 9.5 | 18 | 25.5 | 32 | 37.5 | 42 | 45.5 | 48 | 49.5 | **50** |
| 10.5 | 9 | 17 | 24 | 30 | 35 | 39 | 42 | 44 | **45** | **45** |
| 11 | 8.5 | 16 | 22.5 | 28 | 32.5 | 36 | 38.5 | 40 | **40.5** | 40 |
| 11.5 | 8 | 15 | 21 | 26 | 30 | 33 | 35 | **36** | **36** | 35 |
| 12 | 7.5 | 14 | 19.5 | 24 | 27.5 | 30 | 31.5 | **32** | 31.5 | 30 |
| 12.5 | 7 | 13 | 18 | 22 | 25 | 27 | **28** | **28** | 27 | 25 |
| 13 | 6.5 | 12 | 16.5 | 20 | 22.5 | 24 | **24.5** | 24 | 22.5 | 20 |
| 14 | 5.5 | 10 | 13.5 | 16 | 17.5 | **18** | 17.5 | 16 | 13.5 | 10 |

Once the auction reaches a point where there is no excess demand, i.e, $D_A + D_B \leq 10$, the clock round ends. Suppose this happens when the price per block is 13 and therefore A's final bid is 91 for 7 spectrum blocks.

The clock round bidding behavior can reflect on the bidders' options in the supplementary round in different ways depending on which *cap rule* is imposed on bidders. Cap rule determines the constraints for the supplementary bids imposed by the bids made in the clock round. The most common options are *relative cap rule* and *final cap rule.* The relative cap rule states that if in the supplementary round a bidder wants to increase package size from the clock round, the bid amount is restricted by the bid they made in the last clock round where the eligibility was high enough to bid on the package in question. In other words, say that in round $t$ a bidder reduced their demand from $q_x$ to $q_t$ and now wishes to make a supplementary bid on $q_x$. Then the supplementary bid on $q_x$ is constrained by inequality

$$b(q_x) \leq b(q_t) + (q_x - q_t) \cdot p_t. \tag{6}$$

The idea behind relative cap is that since the bidder reduced her demand in round $t$, she revealed that the maximum she would be willing to pay for the package $q_x$ is her bid in round $t$ plus the difference in package prices between rounds $t$ and the previous round $x$. To better illustrate this, consider the bidding behavior in Table 9 when price increases from 12.5 to 13. When this happens bidder A reduces her demand from 8 to 7 and A's bids for 8 and 7 items are therefore 100 and 91, respectively.

Now if she wishes to bid on 8 items in the supplementary round, the bid amount is constrained by Equation 6. By plugging in the numbers the equation takes form $b(8) \leq 91 + (8 - 7) \cdot 13$, or $b(8) \leq 104$. The result is intuitive: if A was willing to pay more than 104 for 8 items then it would have not made sense for her to reduce her demand from 8 to 7 when the price increased to 13.

Another commonly suggested cap rule is the final cap rule, which states that all supplementary bids are contrained by the packages bid on in the final clock round $f$:

$$b(q) \leq b(q_f) + (q - q_f) \cdot p_f \tag{7}$$

The main difference is that the final cap rule also constrains the bids on smaller packages in the supplementary round. The idea behind this is that it should not make sense for a bidder to signal a higher valuation for a smaller package than she bid on in the final clock round because she had the option of bidding on this smaller package then, but chose not to.

Third, less commonly suggested, cap rule is the *intermediate cap*, which states that the supplementary round bids are restricted by all eligibility-reducing rounds in the clock round, not only by the last round where the eligibility was high enough to bid for the package in question (relative cap) or by the final clock round (final cap).

The most commonly used version is the relative cap[13] and the example presented here will also follow this rule.

So what are A's options for bidding in the supplementary round? First of all, the revealed preference activity rule states that the bidders are not allowed to decrease their bids for packages they previously bid on (they have already signaled that they value the package at certain value, so they can not now bid less than that signaled valuation), therefore the bidding curve $b_A(x)$ sets the lower boundary for A's supplementary round bids. Second of all, A's supplementary bids can not signal a higher marginal value than her original valuation curve. If we call the upper boundary of A's bids $S_A(x)$, then $S'_A(x) \leq V'_A(x)$ must apply. In other words, $S_A(x)$ rises at an equal rate to $V_A(x)$, which means that the distance between $V_A(x)$ and $S_A(x)$ is constant at $V_A(7)$ - $b_A(7)$. The equation for $S_A(x)$ thus is

$$S_A(x) = 20x - \frac{1}{2}x^2 - (115.5 - 91) = 20x - \frac{1}{2}x^2 - 24.5. \tag{8}$$

Figure 4 summarizes the analysis. The area between $S_A(x)$ and $b_A(x)$ shows A's allowed

[13]Relative cap rule was used in, e.g., Switzerland (2012), Ireland (2012), and UK (2013).

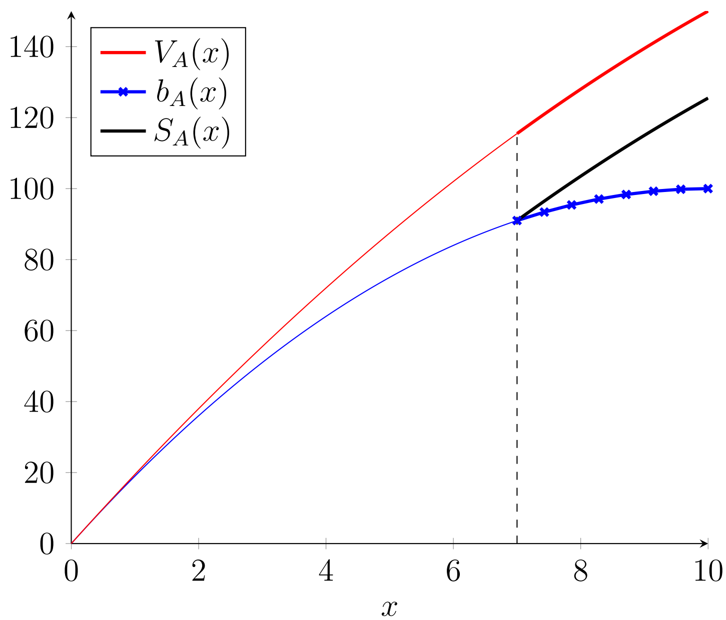


Figure 4: Curves $S_A(x)$ and $b_A(x)$ define bidder A's options for supplementary round bids.

bids in the supplementary round subject to the activity rules. For $x < 7$ A is allowed to bid any non-negative amount as she has not revealed her preference to those packages. Similarly, for $x = 7$ A is allowed to increase her bid because at this point she has only signaled that she is willing to pay *at least* 91 to that package.

Naturally the revealed preference rule becomes much more complicated when licenses with varying values or licenses for different geographical regions are auctioned at the same time, but the fundamental mechanism is the same: By always bidding for the most valuable package throughout the clock round, the bidders maintain the most flexibility in the supplementary round.

### 5.2.2 Pricing rules

When determining the price the winner of the auction has to pay, most auction formats rely on either first-price (winner pays their own bid) or second-price (price is the highest losing bid) rule. However, in CCAs neither of these pricing rules is applicable. Ausubel et. al. (2014) show that the use of first-price rule in a uniform price auction with multiple units can encourage bidders to reduce demand, therefore leading to inefficiencies in the clock round. With Vicrey-pricing, the dominant strategy is to bid the true valuation and the winners win the incremental value created by their bids (i.e., if the winning bid is 50 higher than the second highest bid, the winning bid increased the value of the item by 50, but the price the winner pays is 50 less than their bid). However, when complementarities are present, Vicrey-prices may easily become too low and lead to inefficiencies. Therefore, a more complicated pricing mechanism is required and one of the most prominent ones is called Vickrey-nearest-core pricing, which was proposed by Cramton (2013).

**Vickrey-nearest-core pricing**

Vickrey-nearest-core pricing rule is a modification of Vickrey-pricing that takes into account the constraints introduced by complementarities. The model is best described with a simple example. First consider a situation where two products A and B are auctioned among three bidders B1, B2, and B3. Suppose their valuations $V_i$ for the products are:

$$V_A(B1) = 4, V_B(B2) = 4, V_A(B3) = V_B(B3) = 1.$$

In this case it would be feasible to use Vickrey pricing and the items would be sold to bidders B1 and B2 at a price of 1 per item. Figure 5 illustrates the pricing solution graphically.

Let us now introduce a fourth bidder B4 whose valuation for each individual item is zero, but for the bundle A+B her valuation is 3. The Vickrey solution is still the same, to sell A and B to bidders B1 and B2, respectively, and the total revenue is still 2. However, in this case bidder B4 would have a reason to be upset as she offered to pay 3 for the combination of A + B, which is more than the revenue auctioneer collected from B1 and B2. Clearly the use of Vickrey-pricing in this case is not efficient and this is caused by complementarities between the products. In this case the prices are said to not be in the core as some combination of bidders would have been willing to pay more for the products than what the winners paid. Figure 6 illustrates the situation. The core in this case is smaller as the points within the red triangle are not efficient

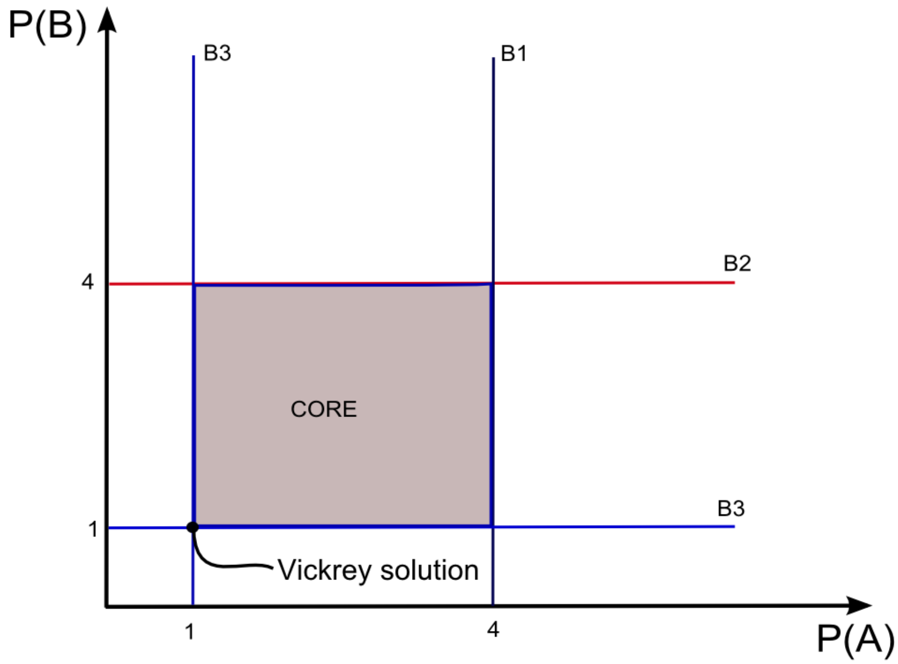


Figure 5: Pricing problem with substitutes. Vickrey-prices lie in the south-west corner of the core.

solutions to the pricing problem.

To solve this, one or both of the payments must be increased in order to find a combination of prices for A and B that support efficient allocation (B1 wins A and B2 wins B) and leave no bidders upset. As the name of the pricing model suggests, the point closest to the Vicrey-prices is chosen, which in this case is the point $P^* = (1.5, 1.5)$, meaning that the items A and B are sold to bidders B1 and B2 at a price of 1.5 each. It is important to note that all points on the south-west facing border of the core would result in the same revenue for the seller (i.e., would be bidder-optimal), but the point with the lowest distance from the Vicrey-prices is the fairest solution.

This particular example is rather easy to solve, but when auctioning multiple spectrum blocks at the same time to bidders with different complementary valuations, the final allocation must be solved using a computer algorithm. This problem of solving the efficient allocation given a set of bids is called a winner determination problem.

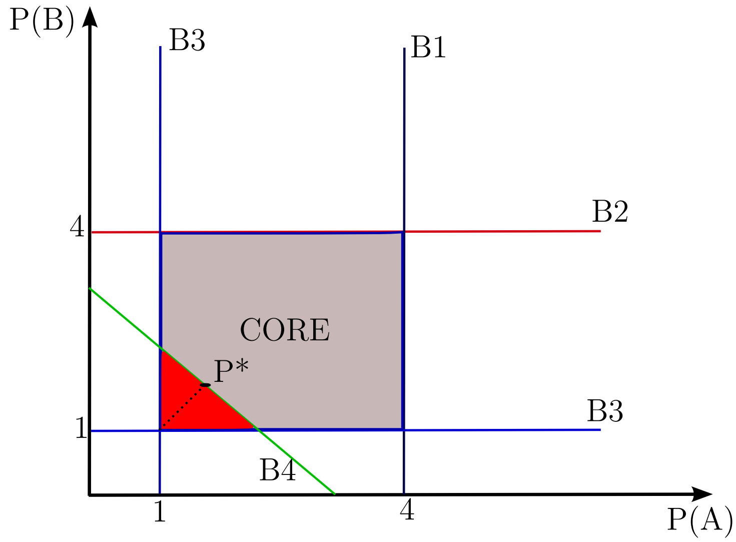


Figure 6: Pricing problem with complementarities. Vickrey-prices are outside of the core.

## 5.3 Winner determination problem

Winner determinations problem (WDP) is a combinatorial optimization problem that must be solved to find the revenue-maximizing solution subject to the constraints created by the activity and pricing rules. All bids from the auction are submitted into the WDP and allocations violating these constraints are eliminated one by one. The most important characteristic of the WDP is the bidding language, which determines how individual bids are treated. CCAs have traditionally used a fully expressive Exclusive-OR (XOR) bidding language that treats all bids from the clock around and the supplementary round as mutually exclusive, all-or-nothing bids. In practice this means that when a bidder submits multiple package bids throughout the auction, the winner determination algorithm considers only solutions where each bidder can have a maximum of one winning bid. For example if a bidder submits package bids {A1, A2, A3} and {B1, B2, B3}, they can only ever win one of these packages and never both of them. While XOR language allows bidders to express their valuation for any package, as the number of licenses increases the WDP becomes more and more complex. Going into the details of the programmatic side of WDPs would be out of the scope of this paper[14], but issues

[14]For more information about WDPs in combinatorial auctions, see, e.g., (Lehmann et al., 2005).

created by bidding languages will be discussed further in Chapter 5.5.

## 5.4 Advantages of CCAs

By far the biggest advantage of CCA compared to SMRA is the model's ability to work efficiently with complementary items and therefore eliminating the exposure risk. By allowing participants to bid on arbitrary packages instead of individual items, the bidders win either the whole package or nothing at all, meaning that they do not have to worry about winning only some of the items and overpaying for them. Moreover, this also shifts the burden of bundling the items appropriately from the auctioneer to the bidders who often have more information regarding the complementary nature of the items. Furthermore, this makes CCA a technology-neutral auction model: In some cases there may be competing technologies that require different amounts of spectrum, which would make efficient bundling before the auction especially difficult.

Carefully planned activity and pricing rules encourage participants to bid truthfully throughout the auction. Truthful bidding being the optimal strategy is of utmost importance, because it makes the auction strategically simple regardless of the otherwise complex rule set. In a CCA with relative cap activity rule, by bidding on the most profitable package throughout the clock round, the bidders maximize flexibility in the supplementary round. Truthful bidding is further encouraged by using a payment-minimizing Vickrey-nearest-core pricing rule when solving the final allocation of licenses. Truthful bidding, in turn, enhances price discovery: The marginal value of products can be derived from the bidding behavior. Price discovery is important especially in spectrum auctions because the real value of the spectrum is difficult to determine. The importance of price discovery is further magnified in an auction with multiple items for sale as the valuation process gets much more complicated. The revealed-preference activity rule also removes the incentives for bidders to reduce demand in the clock round as that would limit their options in the supplementary round. Additionally, the revealed-preference activity rule also helps to reduce other collusive agreements and bid sniping opportunities.

## 5.5 Potential issues in CCAs

While the combinatorial clock auction model successfully eliminates many of the issues associated with SMRA model, many potential problems still exist. Moreover, many of the benefits are theoretical and rely on assumptions such as independent valuation models and bidders understanding and following the optimal strategies, but empirical results show that these assumptions are not always valid. Kroemer et. al. (2016) study the bidding behavior in several combinatorial clock auctions and find that the bidding behavior in reality is often not turthful, i.e., bidders do not bid according to a profit-maximizing strategy.

Untruthful bidding behavior may be a result of bidders' willingness to drive up competitors' prices. Janssen and Karamychev (2013) discuss different strategies in a CCA with heterogenous items under the assumption that bidders do not only care about their own payments but also their rivals' payments. They show that under the final cap rule in the supplementary round, bidders have the option to place high bids on the packages they are not truly interested in with no risk of actually having to pay for those packages, and that way they can drive up the prices their competitors must pay for the licenses. Knapek and Wambach (2013) further discuss spiteful bidding in CCAs and show that in an auction with two bidders B1 and B2, truthful bidding is not a dominant strategy for B1 in the clock round if there is a threat of spiteful bidding from B2, even if B1 herself is only interested in her own payment. They also show that if a relative cap rule is imposed, the allocation can change in the supplementary round even if there was no excess supply after the final clock round. Ausubel and Baranov (2017) discuss this same property of the relative cap rule and show that in order to secure the allocation a bidder won in the clock round, the supplementary bid for the same package might have to be several times larger than the final bid in the clock phase. Truthful bidding is one of the most important features that was hoped carry on to the CCA from a static Vickrey auction, but if it is not a dominant strategy, the difficulty of decision-making throughout the auction is increased significantly.

Levin and Skrzypacz (2016) discuss how demand expansion in the clock phase may be used to relax restrictions in the supplementary round. Consider a situation where two bidders B1 and B2 who have similar linear valuations are participating in an auction for one divisible unit of spectrum. If both bid truthfully throughout the clock phase their demands are always equal and the market clearing is reached when $D_{B1} = D_{B2} = \frac{1}{2}S$ at price $p = p^*$. However, suppose that B1, instead of consistently lowering her demand, holds her demand at $D_{B1} = S$ until price is just below $p^*$ and then drops her demand to $D_{B1} = \frac{1}{2}S$. The final result of the clock phase is the same as before but behaving inconsistently like this B1 has secured much more relaxed boundaries for her bids in the supplementary round. Had B2 anticipated B1's strategy of demand expansion in the clock phase, the optimal response for her would have been to reduce demand early in the auction to reach market clearing leading to an inefficient outcome. It is worth noting that even if B2 reduces her demand, B1 would still obtain an advantage for the supplementary round.

Other possible reasons for inconsistent bidding behavior are budget constraints imposed on bidders by the management. These budget constraints may prevent bidders from placing optimal bids and therefore create inconsistencies in the bidding behavior. While placing bids exceeding the budget is relatively safe in a CCA, meaning that there is very little risk of actually having to make these payments, this aspect of the auction might be hard to explain it to the board of directors and shareholders. Therefore bidders may be forced to move their bids to cheaper packages even though bidding on a larger, more expensive package would be more profitable.(Janssen and Karamychev, 2013; Knapek and Wambach, 2013)

Another potential source for inefficient auction outcome is a large number of potential packages available to bidders leading to a phenomenon referred to as *missing bids*. Bichler et. al. (2014) show that CCA may lead to low revenues if the number of licenses is large. When the number of licenses increases, the number of possible packages increases exponentially, which leads to a situation where only a fraction of possible packages are actually bid on in the supplementary round. For example, in a UK spectrum auction in 2013 bidders submitted between 8 and 62 package bids in the supplementary round, but total number of possible packages was 750. It is unlikely that the bidders had zero-valuation for packages they did not bid on but they are treated as such when solving the optimal allocation. The issue of missing bids was also noticed in, e.g., two of the UK spectrum auctions in 2008 where the number of licenses resulted in tens of thousands of potential packages. When the number of potential bundles is this large, it is easy to see that even if the bidders fully understand the auction model and the optimal strategy, it is impractical to place hundreds or even thousands bids in the supplementary round. Missing bids become an even more severe issue when licenses for several frequency bands are auctioned at the same time as this can increase the number of potential packages to millions. (Bichler et al., 2013)

Missing bids issues is largely created by the XOR bidding language used in the winner determination problem in CCAs. While a fully expressive bidding language sounds attractive, in reality the number of potential packages is often so large that it is not feasible for bidders to communicate all possible valuations even if they acknowledged them. To illustrate this, consider a situation where there are five items a bidder is interested in the supplementary round and she has a positive marginal value for any combination of items. In this case the bidder would have to submit individual bids for any individual item, any pair of items, any three items, any four items, and for the whole package of five items totaling

$$\binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 5 + 10 + 10 + 5 + 1 = 31$$

bids.

While 31 bids is hardy impossible, with the introduction of regional licenses and multiple frequency bands the number quickly grows unfeasible. Additionally, regulators are often forced to limit the maximum number of bids in order to keep the dimensions of the winner determination problem manageable, which means that, in theory, bidders may not be able to express all the valuation they would like to do so.

Missing bids is especially worrisome for the auctioneer as it can easily lead to unallocated licenses at the end of the auction despite seemingly sufficient demand. In fact, this has been noticed in some of the past CCAs[15] and similar results were noticed in a lab experiment by ran by Bichler et. al. (2013).

One potential improvement is to allow bidders to place mutually inclusive OR bids in the supplementary round that can be treated as additive to their clock round package. This can significantly reduce the number of required bids in the supplementary round. To illustrate this, consider the previous situation where a bidder is interested in 1-5 items in the supplementary round. With XOR bid language it required 31 separate bids to express all possible combinations of those 5 licenses. With OR bid language it would be enough for the bidder to submit a bid for each of the 5 licenses and they can win any combination of the licenses, reducing the number of bids by 84 %.

The downside of the OR language is that sometimes bidders want to submit mutually exclusive bids. Consider a situation where licenses for two frequency bands A and B are sold in the same auction. An operator might be interested in licenses in both bands, but not at the same time, i.e., she only wants to win licenses in one of the bands. With XOR language she could submit bids such as {A1, A2} and {B1, B2} and she could ever win only one of the packages. With OR language, however, she could theoretically win both of the packages and end up overpaying for one of them. Bichler et. al. (2013) discuss the possibility of using a hybrid language, which they call OR-of-XOR bidding language. In this case the bids within frequency bands are treated as OR bids, while bids across frequency bands are treated as mutually exclusive XOR bids. The argument behind a bidding language like this is that the complementarities rarely reach across

[15] For example in Slovenia in 2014 20 MHz was left un allocated in the 800 MHz band and in the Netherlands in 2010 55 MHz was unsold in the 2.6 GHz band.

frequency bands and therefore there is less need to be able to express complex valuations. They run simulations comparing the XOR and OR-of-XOR languages and find that the latter results in a higher efficiency, while also simplifying the auction process and winner determination problem. This kind of bidding language might be especially useful in markets with regional licenses, e.g., the US or Canada.

The final problem CCA is facing is the complexity of the model. Even if the auctioneer was able to tweak all the rules in a way that it is truly optimal for bidders to bid truthfully regardless of their competitors' actions, it would still be a tough sell for the financiers and board members. This often means that regardless of the model's theoretical soundness, the operators are always required to hire experts to consult in the strategic planning before the auction, requiring plenty of resources that might not be needed if a simpler auction model was used.

## 5.6 Empirical results from combinatorial clock auctions

Combinatorial clock auction model has been used in spectrum auction around the world relatively frequently over the past decade, and it is constantly gaining more and more attention from the regulators. This Chapter takes a look at the results from some of the combinatorial clock auctions organized in European countries during 2010-2015 and compares them to the results obtained with other auction models, mainly SMRA during the same period. This time period is chosen for the analysis because during that time many countries had already adopted CCA model for spectrum auctions, while many other countries still used SMRA and other models to allocate the licenses for same frequencies. The analysis concentrates on allocation of spectrum right on four important frequency ranges: 800 MHz, 900 MHz, 1800 MHz, and 2600 MHz. The next important frequency range is the 700 MHz range, which operators can use to strengthen their 4G plans or develop 5G services (700 MHz range is sometimes referred to as pre-5G frequencies). So far 700 MHz licenses have been sold in, e.g., Germany (2015), France (2015), Finland (2016), Italy (2018), and Sweden (2018). However, none of the European countries have allocated these licenses using a CCA model and therefore these licenses are omitted from the analysis here. However, this would be a logical next step as several countries are expected to sell the 700 MHz licenses together with licenses for other frequencies suitable for 5G applications and at least Ofcom (The UK regulator) is planning to do so using a CCA model.(Ofcom, 2018)

Validating an auction model for spectrum auctions can be extremely difficult for several reasons. First of all, the end goals are relatively complicated as opposed to, e.g., a single-item auction ran by a revenue-maximizing seller. A complete analysis would require for example post-auction analysis of the telecommunications market. Since this would be vastly out of the scope of this paper, this chapter concentrates on revenue-based metrics of auctions. Second of all, comparing two different spectrum auctions within a country, e.g., SMRA in 2010 and CCA in 2013, is not feasible because the spectrum valuations change. Even if one of the auctions resulted in higher revenue it would not say much about the efficiency of the models if the true value of spectrum is unknown. Third of all, comparing the sale of same frequency bands in different countries is often not feasible due to variation in auction rules and legislation across countries, as well as the differences in the market environment. Lastly, lab experiments would require a large amount of resources and might not lead to conclusive results, and field experiments are not feasible due to the size and nature of the problem.

The most common metric used in the literature for evaluating the success of spectrum auctions is price per megahertz per population [Price/MHz·pop], which measures the revenue acquired per megahertz per head of population. For example, (Mochon and Saez, 2017) evaluated the success of CCAs around the world using this metric. The British regulator Ofcom (2015) took a slightly more advanced approach and derived UK equivalent prices by adjusting the auction results from other countries for purchasing power parity (PPP). However, since the objective of this chapter is not to present any definite proof that one model is better than the other, but rather just perform a comparison between CCA and other methods, [Price/MHz·pop] values will be used for simplicity.

Auctions studied here all took place in Europe between 2010 and 2015 and were used to allocate spectrum within four frequency ranges: 800 MHz, 900 MHz, 1800 MHz, and 2600 MHz.[16] A total of 19 auctions are studied, 9 of which were using a CCA model and 10 some other auction model, in most cases SMRA. Tables 10 and 11 summarize the total amount of MHz sold in each frequeny range in each auction.

Table 10: Total MHz sold in each frequency range: SMRA and other models.

| **Country** | **Year(s)** | **800 MHz** | **900 MHz** | **1800 MHz** | **2600 MHz** |
|---|---|---|---|---|---|
| Germany | 2010 | 60 | - | 50 | 190 |
| Greece | 2011 | - | 70 | 40 | - |
| Italy | 2011 | 60 | - | 30 | 150 |
| Portugal | 2011 | 60 | 10 | 84 | 145 |
| Spain | 2011 | 60 | 10 | - | 140 |
| Sweden | 2011 | 60 | - | 70 | - |
| Romania | 2012 | - | - | - | - |
| Czech Republic | 2013 | 60 | - | 50 | 190 |
| Greece | 2014 | 60 | - | - | 140 |
| Germany | 2015 | - | 70 | 100 | - |

[16] In some auctions additional frequencies were also auctioned but they are left out of the analysis.

Table 11: Total MHz sold in each frequency range: CCAs.

| **Country** | **Year(s)** | **800 MHz** | **900 MHz** | **1800 MHz** | **2600 MHz** |
|---|---|---|---|---|---|
| Denmark[a] | 2010, 2012 | 60 | - | - | 185 |
| Austria[b] | 2010, 2013 | 60 | 70 | 150 | 190 |
| Ireland | 2012 | 60 | 70 | 150 | - |
| Netherlands | 2012 | 60 | 70 | 140 | 180 |
| Switzerland | 2012 | - | 70 | 140 | 175 |
| Slovakia | 2013 | 60 | - | 60 | 190 |
| Slovenia | 2014 | 60 | 70 | 130 | 185 |

[a]800 MHz licenses allocated in 2012 while others in 2010. 900 MHz and 1800 MHz licenses are left out because incubents were not allowed to bid for those licenses and additionally the license period was shorter than normally.

[b]2600 MHz licenses sold in 2010, others in 2013.

Results of the auctions are summarized in Table 12 showing the total MHz sold, total revenue, and price per MHz per head of population. Ideally the comparison would be made within each frequency range but in many cases the regulators did not reveal the frequency range specific revenue information but only the aggregate revenue collected from the auction. Price information is converted to EUR using the average exchange rate[17] of the auction year and price/MHz·pop is derived using the population census[18] of the auction year.

The results show that the price/MHz·pop was 100 % higher in CCAs compared to auctions that used a different auction formats. As discussed before, revenue-based metrics do not tell the whole story and high revenue does not automatically mean that the auction was successful and efficient. However, since there is no evidence that any of the studied auctions resulted in an uncompetitive end market, revenue should be a relatively accurate measure for successfulness of the auction. Moreover, often when issues such as exposure risk, collusion, and signaling are present, their consequence is abnormally low revenues. Therefore, price/MHz·pop can be considered a reasonably good proxy for

[17]Source: https//www.x-rates.com

[18]Source: www.google.com

successfulness of the auction. Given the large difference in price/MHz·pop between the auction formats, it is safe to say that CCAs seem to work better for spectrum allocation compared to previously popular methods. A more complete analysis would require ex-ante and ex-post analysis of the telecommunications markets in auction countries as well as adjusting the results for GDP and purchasing power. This kind of analysis, however, is out of the scope of this paper.

Table 12: Auction results. Upper part of the table shows includes the results from SMRAs and other models while the lower part shows the results from CCAs.

| **Country** | **Year(s)** | **MHz sold** | **Total revenue (MEUR)** | **Price/MHz·pop (EUR)** |
|---|---|---|---|---|
| Germany | 2010 | 300 | 3937 | 0.16 |
| Greece | 2011 | 110 | 380.5 | 0.31 |
| Italy | 2011 | 240 | 3945.5 | 0.27 |
| Portugal | 2011 | 299 | 374 | 0.12 |
| Spain | 2011 | 210 | 1647 | 0.17 |
| Sweden | 2011 | 130 | 375 | 0.31 |
| Romania | 2012 | 375 | 682.1 | 0.09 |
| Czech Republic | 2013 | 300 | 312 | 0.10 |
| Greece | 2014 | 200 | 381.1 | 0.17 |
| Germany | 2015 | 170 | 969.6 | 0.07 |
| ***Average*** | | | | ***0.18*** |
| Austria | 2010, 2013 | 470 | 2053.5 | 0.52 |
| Denmark | 2010, 2012 | 245 | 234.6 | 0.17 |
| Ireland | 2012 | 280 | 854.6 | 0.66 |
| Netherlands | 2012 | 450 | 3800 | 0.50 |
| Switzerland | 2012 | 385 | 1203 | 0.39 |
| Slovakia | 2013 | 310 | 163.9 | 0.10 |
| Slovenia | 2014 | 445 | 148.8 | 0.16 |
| ***Average*** | | | | ***0.36*** |

# 6 Conclusion

Until the mid 1990s the radio spectrum was considered an abundant resource with only few use cases, and licenses were often awarded using so called beauty contests or lotteries. The problem with beauty contests is the mechanism's opaqueness and vulnerability to corruption and favoritism. While lotteries provide a fairer solution, there is no way of guaranteeing that the spectrum is utilized efficiently. As the demand for spectrum rights started to increase, a new allocation method was required. The first spectrum auction was organized in the US in 1994 and ever since it has been the dominant method in spectrum allocation for governments all over the world.

Auctions have enabled price discovery and help the governments to put the licenses into the hands of the parties that have the highest valuation for the licenses. This is a key element in helping the governments to reach its most important goals of efficient allocation of spectrum rights and promotion of competition in the end market. This in turn encourages the companies to develop the products faster, increasing the technological advancement in the country. Additionally, auctions help governments to collect significant revenues in exchange for the licenses.

Even though auctions have been used in spectrum allocations for a better part of three decades, there is still no generally accepted best solution when it comes to choosing the auction mechanism. This is due to differences between the market environments as well as the differences between individual spectrum blocks. This means that every spectrum auction should be treated as its own problem and auction design should be approached carefully and in great detail. However, some auction mechanisms have grown to be more commonl than others. By far the most commonly used auction mechanism is the simultaneous multiround auction (SMRA) model which consists of several English auctions that are ran concurrently. SMRA has shown good results in many cases and with carefully planned auction rules many shortcomings have been eliminated effectively. The downside of SMRA is that it requires a lot of resources from

the auctioneer as well as from the bidders.
When auctioning homogeneous licenses, a good substitute for SMRA is an ascending clock auction where the seller announces prices and bidders respond by submitting their demand for spectrum rights at the current price. Despite the simplicity and resource-friendliness, ascending clock auction model has not gained traction among auctioneers due to its vulnerability to collusion, and SMRA has dominated the spectrum auction environment.

Over the past decade the advancement of technology and emergence of new applications has increased the complementary nature of spectrum licenses - a feature that sets spectrum auctions apart from most multi-item auctions. Complementarities across products introduces the bidders to exposure risk, which is something SMRA has not been able to handle well. This significantly reduces SMRA's ability to perform well when auctioning licenses and has driven economists to develop alternative auction mechanisms. By far the most prominent candidate has been the combinatorial clock auction (CCA) model which comprises of two stages, clock round and a sealed-bid supplementary round. With carefully planned activity rules CCA model is hoped to encourage bidders to act truthfully throughout the auction helping the governments to allocate the licenses efficiently as well as simplify the strategic planning for bidders. CCA is designed to combine the effectiveness of ascending clock auction model as well as the theoretical soundness of the Vickrey-Clarke-Groves mechanism. The key feature of CCA is that it allows bidders to bid on arbitrary bundles of products eliminating exposure risk completely.

While CCA is by far the most promising auction model for future spectrum auctions, several problems still exist. The biggest issue is the model's reliance on truthful behavior of bidders, while studies have shown that this is often not the case even if it is theoretically optimal. Untruthful behavior is caused by budget constraints and bidders' willingness to affect their competitors' prices. Additionally, as the number of licenses increases the potential number of packages grows exponentially complicating both the bidders' decision making process as well as the optimization problem faced by the seller. Moreover, the model's complexity requires companies to dedicate more resources when planning their strategies for the auction.

Despite the issues still present, the empirical results show that CCA has worked well in the past. It is, however, important to remember that each spectrum auction should be treated as its own problem and auction design should be approached accordingly, without shutting out any of the potential auction mechanisms. While it is safe to say that CCA model is likely to outperform other models in situations where the threat of exposure risk is real, i.e., in auctions where several frequency bands are sold at the same time, especially in fragmented markets with regional licenses, it can easily create unnecessary complexities in simpler environments. Therefore when fewer licenses are auctioned or the licenses are substitutes, an ascending clock auction or a SMRA are likely to produce equally good results but much more effectively.

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